Now, had both started together, and drank simultaneously, they would have consumed the wine skin in two hours less time. And, in this case, Dionysius' share would have been one-half as much as Silenus did secure, by waking and snatching the skin.

In what time would either one of them alone finish the goat-skin?

Solution by PROFESSOR F. L. GRIFFIN, Ph. D., Williams College.

Let x=fractional part which S drank, and y=number of hours S requires for entire skin. Then $\frac{2}{3}y$=time D was drinking, and xy=time S was drinking; $y(\frac{2}{3}+x)$ = time they used consecutively. Also, since $\frac{3(1-x)}{2y}$=part D drinks per hour, or $\frac{3(1-x)+2}{2y}$=part both drink per hour, the time required when drinking simultaneously$=\frac{2y}{5-3x}$.

Hence, (A) $y(\frac{2}{3}+x)=2+\frac{2y}{5-3x}$.

Again, the part D would get when they drink simultaneously $=\left(\frac{2y}{5-3x}\right)\frac{3(1-x)}{2y}$, or $\frac{3-3x}{5-3x}$; hence, by the problem, (B) $\frac{3-3x}{5-3x}=\frac{1}{2}x$.

Equation (B) gives $x=\frac{2}{3}$ or 3, the latter value being impossible.

Then (A) becomes $\frac{4}{3}y=2+\frac{2}{3}y$, or y=3; and since D drinks $\frac{1}{6}$ per hour, his time would be 6 hours.

Also solved by V. M. Spunar, G. B. M. Zerr, and J. Scheffer.

318. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

Sum to infinity the series $n/(4n^2-1)^2$ beginning with n=1.

Solution by J. W. CLAWSON, Ursinus College, Collegeville, Pa.; HOWARD C. FEEMSTER, A. B., York College, York, Neb.; J. EDWARD SANDERS, Weather Bureau, Chicago, Ill., and S. LEFSEHETZ, Pittsburg, Pa.

$$\sum_{n=1}^{n=\infty}\frac{n}{(4n^2-1)^2}=\sum_{n=1}^{n=\infty}\tfrac{1}{8}\left[\frac{1}{(2n-1)^2}-\frac{1}{(2n+1)^2}\right]$$

$$=\underset{n=\infty}{\text{Lt.}}\tfrac{1}{8}\left[\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots\frac{1}{(2n-1)^2}\right)\right.$$

$$\left.-\left(\frac{1}{3^2}+\frac{1}{5^2}+\dots\frac{1}{(2n-1)^2}+\frac{1}{(2n-1)^2}\right)\right]=\underset{n=\infty}{\text{Lt.}}\tfrac{1}{8}\left[\frac{1}{1^2}-\frac{1}{(2n+1)^2}\right]=\tfrac{1}{8}.$$

Also solved by V. M. Spunar, G. B. M. Zerr, J. Scheffer, S. A. Corey, and T. J. Fitzpatrick.

GEOMETRY.

343. Proposed by O. J. BROWN, Fairhope, Ala.

From any external point of a triangle, to draw a line so as to divide the triangle into two equal parts.

I. Solution by C. N. SCHMALL, New York City.

Construction: Let P be the given point and ABC the given triangle. Join D, E, F, the middle points of the sides. Now draw PGI parallel to BC and join EG. From D draw DH parallel to GE, and then draw HI parallel to BA and meeting PG prolonged in I. On PI as a diameter describe a semi-circle and thereon lay off $PK=PG$. Draw KI, and in the base BC lay of $HM=KI$. Draw PM cutting AB in L. The line PLM bisects the triangle and is the line required.

Proof: Let PM and HI meet in N. Now the triangles PGL, PIN, MHN are clearly similar.

Also, since $PI^2=PK^2+KI^2=PG^2+HM^2$ (for $PK=PG$, and $KI=HM$).

$\therefore \triangle PIN=\triangle PGL+\triangle MHN$...(1).

Hence, the quadrilateral $LGIN$ is equal to the triangle MHN;

$\therefore$ triangle BML=parallelogram $BHIG$...(2).

Again, GE and DH are parallel;

$\therefore GB : BE = DB : BH$...(3).

Hence, the parallelograms $BHIG$ and $BEFD$ have a common angle B, and the including sides are reciprocally proportional.

$\therefore$ by (3), parallelogram $BHIG$=quadrilateral $BEFD$.

But by (2), parallelogram $BHIG$=triangle BML...(2).

$\therefore$ triangle BML=parallelogram $BEFD=\frac{1}{2}$ triangle ABC.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let P be the given point. Draw PE parallel to AB, cutting AC in D; make parallelogram $DEFA=\frac{1}{2}$ triangle ABC. On AB at F erect perpendicular $FG=PD$, and make $GA=PE$. Connect P with Q, then PQ will be the required line which bisects triangle ABC.

For, $\triangle FHQ : \triangle PHE=FQ^2 : PE^2=PE^2-PD^2 : PE^2$.

$$\therefore \frac{\triangle FHQ}{\triangle PHE}=1-\frac{PD^2}{PE^2}=1-\frac{\triangle PDH}{\triangle PEH}.$$

$\therefore \triangle FHQ=\triangle PEH-\triangle PDI=DIHE$; $\therefore FHQ+IHFQ=DIHE+IHFA=DEFA$.

Note: To construct $DEFA$, connect C with the mid-point M of AB, draw DM, and CN parallel to DM, then F will be the mid-point of AN.

Also solved by G. B. M. Zerr, V. M. Spunar, and Daniel B. Northrup.

NOTE. The following gentlemen should have received credit for solving 342: J. A. Caparo, V. M. Spunar, and S. Lefsehetz.

CALCULUS.

274. Proposed by J. EDWARD SANDERS, Weather Bureau, Chicago, Ill.

About the vertices of a regular tetrahedron four spheres are drawn with radii equal to the edge of the tetrahedron. Find the volume common to them all.